

Carine Senior High School

Year 12 Semester Two Examination, 2023

Question/Answer booklet

MATHEMATICS  
SPECIALIST  
UNITS 3&4

If required by your examination administrator, please place your student identification label in this box

Section Two:  
Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 7 | 7 | 50 | 48 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 90 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

|  |  |  |
| --- | --- | --- |
| Markers use only | | |
| Question | Maximum | Mark |
| 8 | 6 |  |
| 9 | 9 |  |
| 10 | 6 |  |
| 11 | 7 |  |
| 12 | 7 |  |
| 13 | 9 |  |
| 14 | 7 |  |
| 15 | 7 |  |
| 16 | 8 |  |
| 17 | 6 |  |
| 18 | 9 |  |
| 19 | 9 |  |
| Section 2 Total | 90 |  |
| Weighted (×0.7222) | 65% |  |

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (90 Marks)

This section has**twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 (6 marks)

A small helium balloon is released and rises vertically so that its height metres above its launch site after seconds is given by . A video camera is located metres horizontally from the launch site of the balloon and automatically rotates so that it is always pointing directly at the balloon.

Determine the rate at which the camera is rotating seconds after the balloon is released.

Question 9 (9 marks)

<EFOFEX>
id:fxd{d1a88875-eb2f-4c7d-ac86-281b86479acb}

FXData:
</EFOFEX>The complex numbers and are  
shown in the complex plane at right.

(a) Express in polar form. (1 mark)

(b) Express in Cartesian form. (1 mark)

(c) Plot in the complex plane above, given that . (2 marks)

(d) Determine the argument of when . (2 marks)

(e) Let . Express in polar form in terms of the real constants and .

(3 marks)

Question 10 (6 marks)

The graph of is shown.

<EFOFEX>
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Using the set of axes provided, draw the graph of

(a) . (2 marks)

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(b) . (2 marks)

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(c) . (2 marks)

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Question 11 (7 marks)

A factory advertises that its tea light candles burn for an average of minutes. The standard deviation of the burn times is known to be minutes.

(a) Quality control took a random sample of candles from the factory production line and recorded their burn times. These times were used to calculate the percent confidence interval for the population mean burn time as minutes. Determine the value of . (3 marks)

A consumer watchdog tested a random sample of candles made by the factory and their mean burn time was minutes.

(b) Describe and construct a suitable interval estimate based on this sample that can be used to advise the watchdog on the reasonableness of the factory’s advertising and use the interval estimate to provide that advice. (4 marks)

Question 12 (7 marks)

<EFOFEX>
id:fxd{edf54efd-7f3f-4cd5-b6a4-f139dd2de51c}

FXData:
</EFOFEX>The slope field for the differential equation

where is a constant, is shown at right.

(a) Use a feature of the slope field to explain why and hence determine the slope at the  
point . (2 marks)

(b) Determine the solution of the differential equation that contains the point in the form . (4 marks)

(c) Sketch the solution curve that contains the point on the slope field. (1 mark)

Question 13 (9 marks)

The Cartesian equation of sphere is .

(a) State the vector equation of sphere . (1 mark)

The position vector of particle at time seconds is given by .

(b) Show that the path of is tangential to sphere . (3 marks)

Particle is moving with a constant velocity and has position vector when . Three seconds later, its position vector is .

(c) Show that the paths of and cross but that they do not collide. (5 marks)

Question 14 (7 marks)

(a) Show that . (1 mark)

(b) Hence show that , using the substitution  
 where appropriate. (6 marks)

Question 15 (7 marks)

<EFOFEX>
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FXData:
</EFOFEX>(a) Use the substitution to show that ,  
where is a constant of integration. (4 marks)

(b) The equation of the curve shown is  
  
.

Determine the area enclosed by the  
curve and the line . (3 marks)

Question 16 (8 marks)

A machine fills bags with salt. The mean and standard deviation of the weight of salt it delivers into a bag is and grams respectively. An inspector routinely takes a random sample of bags filled by the machine.

(a) For repeated random sampling of bags of salt filled by this machine, state the approximate distribution of the sample mean that the inspector should expect. (3 marks)

(b) Determine the probability that the mean weight of a random sample of bags of salt is less than grams, given that the sample mean is greater than grams. (2 marks)

(c) Occasionally, the inspector only has enough time to take a random sample of bags. In the long run, of sample means derived from samples with this smaller size will lie in the range grams. Determine the value of . (3 marks)

Question 17 (6 marks)

Consider the function , where and are positive constants.

The graph of cuts the -axis at , has a horizontal asymptote with equation and has as one of its vertical asymptotes.

(a) Determine . (3 marks)

(b) Now consider the graph of . State the

(i) equation of its horizontal asymptote. (1 mark)

(ii) -axis intercepts. (1 mark)

(iii) equations of its vertical asymptotes. (1 mark)

Question 18 (9 marks)

A particle is moving and has position vector metres, where is the time in seconds since motion began. Its path is shown in the diagram below.

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</EFOFEX>

(a) Mark point on the diagram above to show the position of the particle when , and state the time taken for the particle to next return to this position. (2 marks)

(b) Determine the velocity of the particle when . (2 marks)

(c) Determine the distance moved by the particle during its third second of motion.

(2 marks)

(d) Using the identity , or otherwise, determine the Cartesian equation of the path of the particle. (3 marks)

Question 19 (9 marks)

(a) Plot the complex number that satisfies the conditions and on the Argand diagram below. (2 marks)

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</EFOFEX>

(b) Let and be another complex number. The locus of a complex number satisfies the condition and is shown in the diagram below.

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</EFOFEX>

(i) Determine the complex number . (2 marks)

(ii) On the same diagram, indicate the locus of a complex number that satisfies the condition . (1 mark)

(c) The locus of points that satisfy is an arc of a circle.

(i) Sketch the locus of in the complex plane. (2 marks)

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(ii) Determine, with justification, the exact location of the centre of the circle. (2 marks)

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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